

Number patterns: Linear patterns

The difference between consecutive terms is constant

Patterns can be represented by:

- numbers: 4; 7; 10; ...
- symbols: $T_n = 3n + 1$
- pictures
- tables

Three strategies to determine the general term of the row:
2; 5; 8; 11; ...

Analyse the structure

$$\begin{array}{cccc} T_1 & ; & T_2 & ; & T_3 & ; & T_4 \\ 2 & ; & 5 & ; & 8 & ; & 11 \\ 2 & ; & 2 + 1(3) & ; & 2 + 2(3) & ; & 2 + 3(3) \end{array}$$

$$\begin{aligned} \therefore T_n &= 2 + (n-1)(3) \\ &= 2 + 3n - 3 \\ &= 3n - 1 \end{aligned}$$

$$T_n = m \cdot n + c$$

$m = \text{constant difference} = 3$

$$\begin{aligned} \therefore T_n &= m \cdot n + c \\ \therefore T_n &= 3n + c \end{aligned}$$

Substitute $T_1 = 2$ in: $T_n = 3n + c$

$$\begin{aligned} \therefore 2 &= 3(1) + c \\ \therefore c &= -1 \\ \therefore T_n &= 3n - 1 \end{aligned}$$

$$T_n = a + (n-1)d$$

$a = \text{first term} = 2$

$d = \text{constant first difference}$
 $= T_2 - T_1 = 5 - 2 = 3$

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 2 + (n-1)(3) \\ &= 2 + 3n - 3 \\ &= 3n - 1 \end{aligned}$$

Example: Number patterns

1. a) Determine the general term: 3; 7; 11; ...

Method 1: Analyse the structure.

T_1	T_2	T_3	T_4	T_n
3	7	11	15	
3	3 + 1 (4)	3 + 2 (4)	3 + 3 (4)	3 + (n-1) (4)

The bold number in the third row is 1 less than number of the term.

$$\therefore T_n = 3 + (n - 1)(4)$$

$$\therefore T_n = 3 + 4n - 4$$

$$\therefore T_n = 4n - 1$$

Method 2: First difference is constant: $T_n = m(n) + c$

$$3 \xrightarrow{7-3=4} 7 \xrightarrow{11-7=4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4}$$

$$\therefore \text{First difference is constant: } T_2 - T_1 = 4 \text{ and } T_3 - T_2 = 4$$

$$\therefore \text{Linear: } T_n = m(n) + c$$

$$m = \text{constant difference} = 4; \text{ therefore } T_n = 4(n) + c$$

Substitute first term back in to determine n and c :

$$T_1 = 3; \text{ therefore } n = 1 \text{ and } T_n = 3$$

$$\therefore 3 = 4(1) + c$$

$$\therefore c = -1$$

$$\therefore T_n = 4n - 1$$

Method 3: First difference is constant \Rightarrow linear pattern: $T_n = a + (n - 1)d$

$$3 \xrightarrow{7-3=4} 7 \xrightarrow{11-7=4} 11 \xrightarrow{+4} 15 \xrightarrow{+4} 19 \xrightarrow{+4}$$

$$\therefore a = 3$$

① First term

$$\therefore d = 4$$

② Constant difference

$$\therefore T_n = 3 + (n - 1)(4)$$

③ Substitute in: $T_n = a + (n - 1)d$

$$\therefore T_n = 3 + 4n - 4$$

④ Simplify

$$\therefore T_n = 4n - 1$$

- b) Determine the 100th term of the sequence: 3; 7; 11; ...

$$\therefore T_n = 4n - 1$$

① General term (See question 1a.)

$$\therefore T_{100} = 4(100) - 1$$

② Substitute: $n = 100$

$$\therefore T_{100} = 399$$

- c) Determine which term in the sequence 3; 7; 11; ... has a value of 199.

$$\therefore T_n = 4n - 1$$

① General term (See question 1a.)

$$\therefore 199 = 4n - 1$$

② $T_n = 199$

$$\therefore 200 = 4n$$

③ Determine n

$$\therefore n = 50$$

Worksheet 9: Number patterns

1. Determine the first four terms in each pattern:

- a) $T_n = 3n - 4$
- b) $T_n = -2n + 3$
- c) $T_n = 4$
- d) $T_n = 2^n$
- e) $T_n = n$
- f) $T_n = \frac{1}{2}(n - 6)$

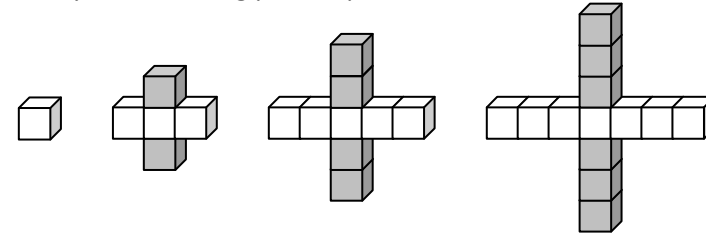
2. Determine the next three terms in each pattern:

- a) 3; 7; 11; 15; ...
- b) 3; 0; -3; -6; ...
- c) 2; -4; 8; -16; ...
- d) 1; 4; 9; 16; ...

3. Determine the general term as well as the 20th term in each pattern:

- a) 3; 7; 11; 15; ...
- b) 3; 0; -3; -6; ...
- c) -6; -2; 2; ...
- d) -50; 0; 50; ...
- e) 4; 9; 14; ...
- f) -8; -10; -12; ...

4. Study the following picture pattern:



- a) Write down an expression that describes the relationship between the pattern number and the number of white cubes.
- b) Write down an expression that describes the relationship between the pattern number and the total number of cubes.
- c) Determine the number of white cubes in the 30th pattern.

5. Each of the numbers 333, 2 106, 13 806, 81 and 27 is divisible by 9.

a) Study the following table and come to a conclusion .

Number	Sum of the numbers
333	9
2 106	9
13 806	9
81	9
27	9

b) Show that your conclusion is also valid for the number made up of the letters $abcd$.

6. Determine the next six terms in each pattern:

- a) 2; a ; 6; b ; 10; c ; ...
- b) $\frac{2}{2}$; $\frac{4}{5}$; $\frac{6}{8}$; $\frac{8}{11}$; ...

Worksheet 9: Number patterns (p. 27)

1. a) $-1; 2; 5; 8$
b) $1; -1; -3; -5$
c) $4; 4; 4; 4$
d) $2; 4; 8; 16$
e) $1; 2; 3; 4$
f) $-\frac{5}{2}; -2; -\frac{3}{2}; -1$

2. a) $19; 23; 27$
b) $-9; -12; -15$
c) $32; -64; 128$
d) $25; 36; 49$

3. a) $T_n = 3 + 4(n - 1)$
 $T_n = 4n - 1$
 $T_{20} = 4(20) - 1 = 79$

b) $T_n = 3 - 3(n - 1)$
 $T_n = 6 - 3n$
 $T_{20} = 6 - 3(20) = -54$

c) $T_n = -6 + 4(n - 1)$
 $T_n = 4n - 10$
 $T_{20} = 4(20) - 10 = 70$

d) $T_n = -50 + 50(n - 1)$
 $T_n = 50n - 100$
 $T_{20} = 50(20) - 100 = 900$

e) $T_n = 4 + 5(n - 1)$
 $T_n = 5n - 1$
 $T_{20} = 5(20) - 1 = 99$

f) $T_n = -8 - 2(n - 1)$
 $T_n = -2n - 6$
 $T_{20} = -2(20) - 6 = -46$

4. a) $1; 3; 5; 7$
 $T_n = 1 + 2(n - 1)$
 $T_n = 2n - 1$

b) $1; 5; 9; 13$
 $T_n = 1 + 4(n - 1)$
 $T_n = 4n - 3$

c) $T_{30} = 2(30) - 1 = 59$

5. a) If the sum of the digits in a number is 9, the number is divisible by 9.

b) Suppose $a + b + c + d = 9$. Then $d = 9 - a - b - c$.
The value of the number $abcd$ is:
 $1\ 000a + 100b + 10c + d$
 $= 1\ 000a + 100b + 10c + (9 - a - b - c)$
 $= 999a + 99b + 9c + 9$
 $= 9(111a + 11b + c + 1)$
Therefore $abcd$ is divisible by 9.

6. a) $14; d; 18; e; 22; f$
b) $\frac{10}{14}; \frac{12}{17}; \frac{14}{20}; \frac{16}{23}; \frac{18}{26}; \frac{20}{29}$