How far can we see?

Gerrit Stols (gerrit.stols@up.ac.za)
University of Pretoria
Tel: 012 420 5550

Department of Science, Mathematics and Technology Education
Faculty of Education
University of Pretoria
Pretoria
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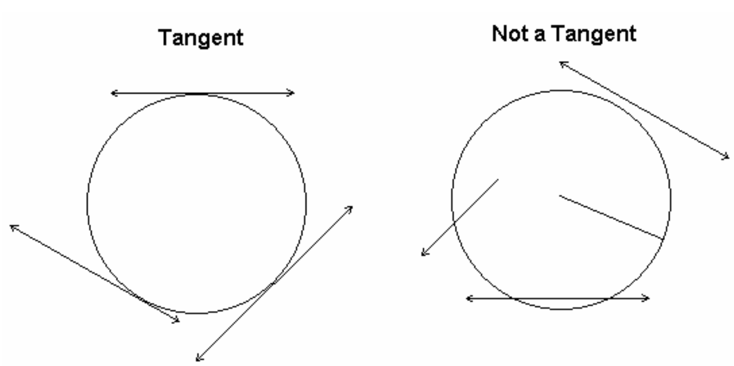
In this paper an example is presented of how tangent properties of circles can be dealt with in context. The emphasis is on mathematical modelling and dealing with real life contexts. This paper wants to explore, use and discover some of these theorems in the context of a boy sitting in a tree on the lookout for cattle.

Context: How far can we see?

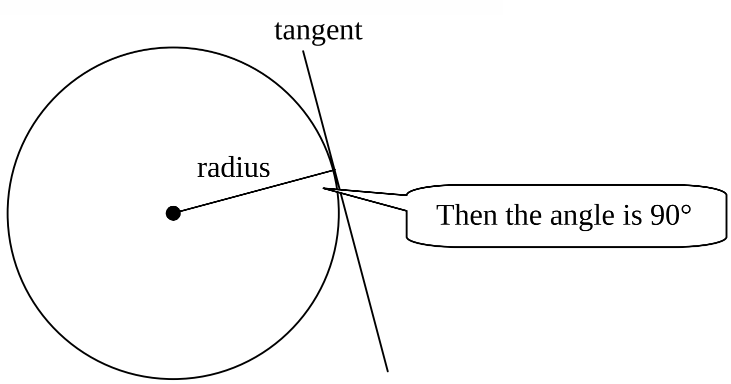
A boy sits in a tree, 30 m high, on the lookout for cattle. How far can he see? To simplify the problem we assume that it is a landscape like the Kalahari without any mountains or any other trees. As he looks out over the field his vision extends to the horizon. Before you continue reading, take a piece of paper and try to solve the problem without help.

The first step to solve the problem is to understand it. A drawing may help you.

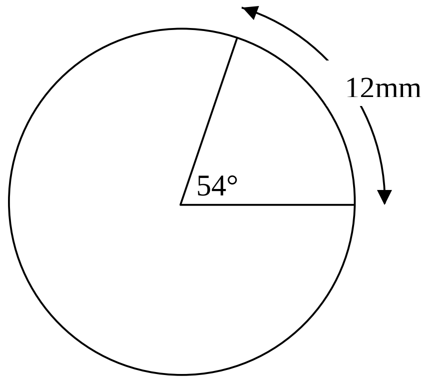
What is the information required in order to solve the problem? You need to know more about tangents. The following activities will help you. Here are some examples and non-examples of tangents.



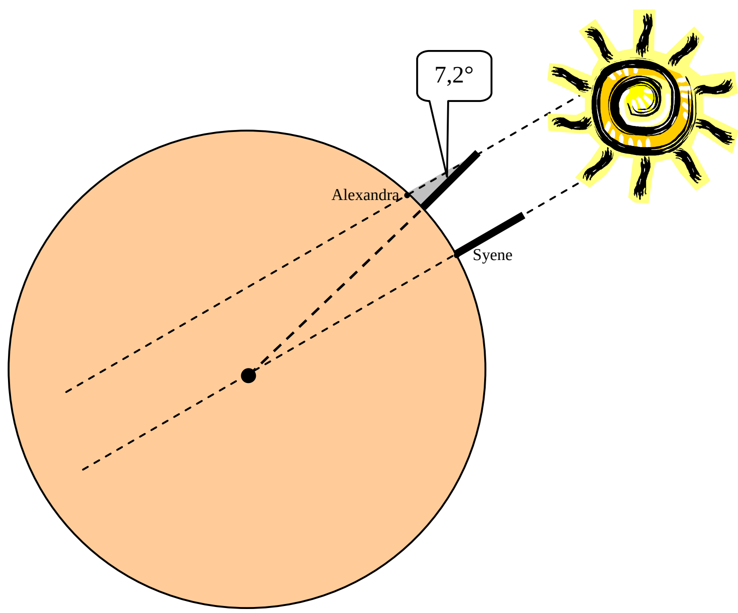
An important property of tangent: a tangent is perpendicular to the radius drawn at the point of contact with the circle.



Activity 1: Find the radius of the accompanying circle:

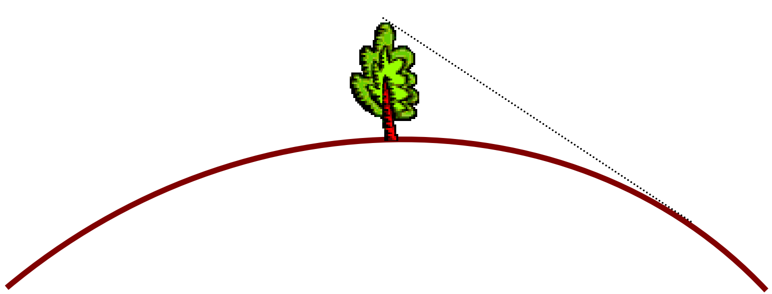


Activity 2: The circumference of the earth can be calculated just by using two sticks. Eratosthenes (276-194 BC) did it by studying shadows! He discovered that at noon on the 21st of June, pillars in the Egyptian town of Syrene cast no shadows. But when he tried the same experiment in Alexandria at the same time of day, a stick held vertically did cast a shadow. Determine the circumference of the earth if the distance between Alexandria and Syrene is 853 kilometers and the shadow of the stick made an angle of 7,2°.



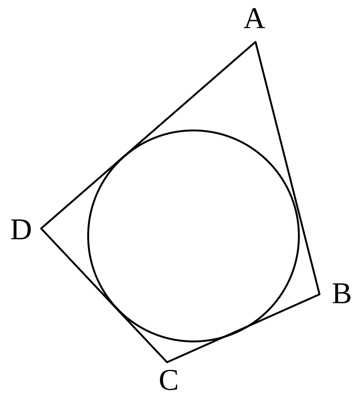
You now have enough information to solve the following activity:

Activity 3: A boy sits in a tree, 30 m high, on the lookout for cattle. How far can he see? Assume that the radius of the earth is 6788 km.

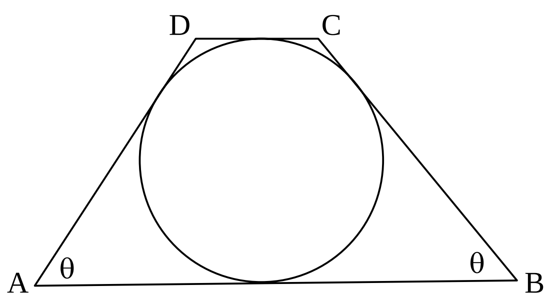


Activity 4: Prove that if two tangents are drawn from the same point outside the circle, then they are equal in length.

Activity 5: Prove that if a quadrilateral ABCD circumscribes a circle, then the sum of the lengths of the pairs of opposite sides is equal, that is AB + CD = BC + DA



Activity 6: The trapezium shown below is circumscribes a circle. The two base angles marked q are equal and sin θ = 0,8. The area of the trapezium is 180 mm². Find the perimeter of the trapezium.



Summary

Although all of us who teach mathematics know the properties of circles, it is important for learners to explore and discover each of these properties by construction. It is better to spend some time on a few good problems than to do a lot of similar routine exercises. It takes time, but it is worthwhile because it creates deep conceptual understanding. I believe that you will experience that mathematical modelling helps learners to deepen their understanding of mathematics.

References

Department of Education. 2003. Mathematics national curriculum statements for grade 10-12 schools. Pretoria: Government Publishers.

The Trustees of Indiana University. 2001. A Moment of Science. http://amos.indiana.edu/library/scripts/stick.html

Appendix: Solutions

Activity 1

If 54° gives an arc of length 12mm, then 1° gives an arc of length 12/54 mm.
Therefore 360° will give an arc of length (12/54) × 360 mm = 80mm.
The circumference of the circle is 80mm.
But the circumference = $2\pi r$

$\therefore$ 80 = 2pr

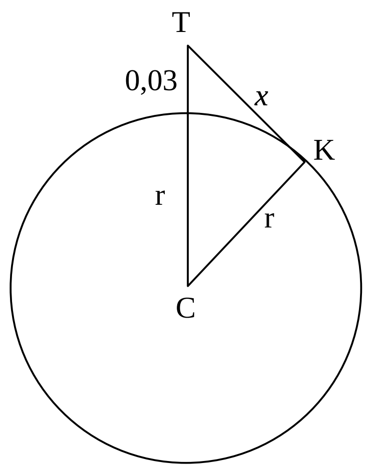
$\therefore$ r = 12,73mm

Activity 2

If 7,2° gives an arc of length 853km, then 1° gives an arc of length 853/7,2 km.
Therefore 360° will give an arc of length (853/7,2) × 360 = 42650km.
The circumference of the earth is 42650km.
But the circumference = $2\pi r$

$\therefore 42650 = 2\pi r$

$\therefore$ r = 6788 km



Activity 3

K = 90° (tangent ⊥ radius)
According to Pythagoras:

$(r + 0{,}03)^2 = r^2 + x^2$

$r^2 + 0{,}06r + 0{,}0009 = r^2 + x^2$

$x^2 = 0{,}06r + 0{,}0009$

But r = 6788km

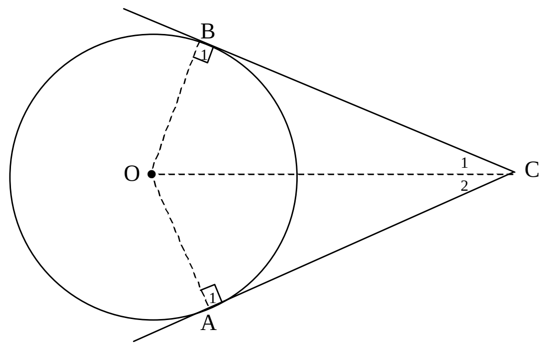
x = 20,18 km

Activity 4

Given: Circle O with tangents BC and AC from point C

Required to proof: BC = AC

Proof: Draw OB, AO and OC



In ΔOBC and OAC:

OC is a common side

OB = OA (radii)

$\hat{B}_1 = \hat{A}_1$ (Assumption: A tangent is perpendicular to the radius)

$\therefore$ ΔOBC ≡ OAC (90°∠, side, hypotenuse)

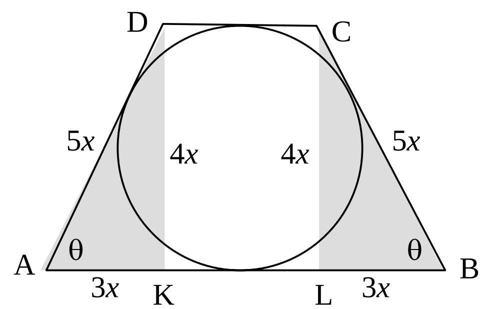
$\therefore$ BC = AC

Activity 5

AK = AN
DK = DL
CL = CM
BM = BN
Then AB + CD

= (AN + BN) + (DL + CL)
= (AK + BM) + (DK + CM)
= (AK + DK) + (BM + MC)
= BC + AD

Activity 6



sin θ = 0,8
sin θ = 4/5
Let DK = 4x then AD = 5x
∴ AK = 3x (Pyth)
Let KL = y
Area = (3x)(4x) + (4x)(y)
180 = $12x^2 + 4xy$ ………… eq 1
AD + BC = DC + AB
10x = 2y + 6x
2x = y ………… eq 2
Substitute 2 into 1: 180 = $12x^2 + 8x^2$
∴ $20x^2 = 180$
∴ x = 3 & y = 6
∴ Perimeter = 2y + 16x = 60mm